

# Measuring progress to predict success: Can a good proof strategy be evolved?

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AITP 2017 – Obergurgl, March 29, 2017

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Things are actually not so dark:

- email me, I can send you an executable
- find one at <https://www.starexec.org/>
- (don't) look for the source at:  
<http://www.cs.miami.edu/~tptp/CASC/J8/Entrants.html>

- 1 The role of strategies in modern ATPs
- 2 Proving with orderings
- 3 How to evolve a precedence?
- 4 Conclusion

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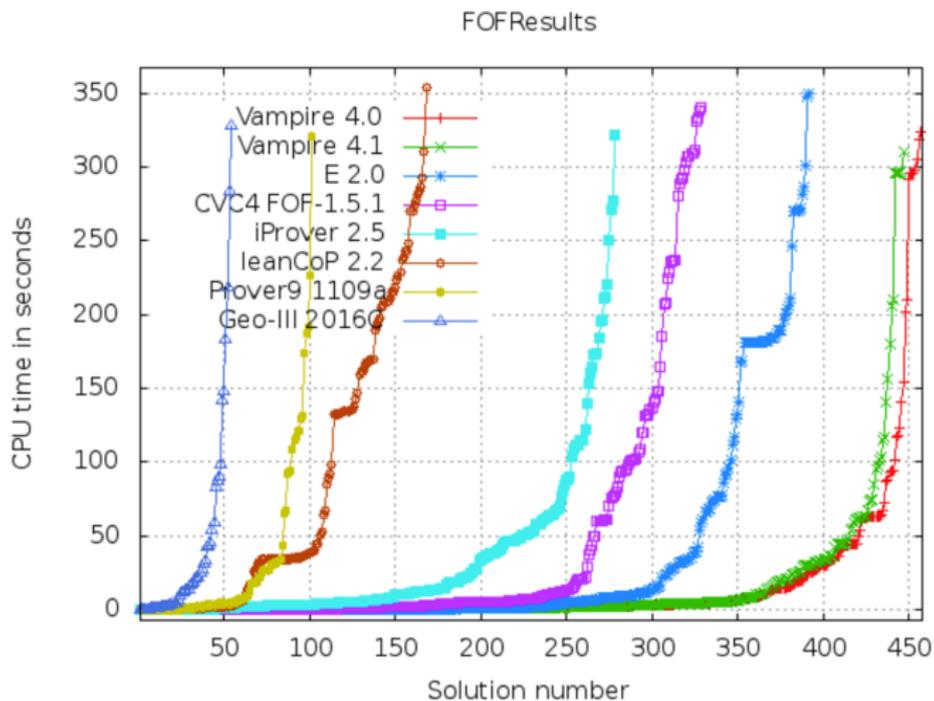
What does this mean?

- There is no single best strategy
- It's usually better to start something else than to wait
- Strategy Scheduling (portfolio approach)

# CASC-mode: a conditional schedule of strategies

```
case Property::FNE:
  if (atoms > 2000) {
    quick.push("dis+1011_40_bs=on:cond=on:gs=on:gsaa=from_current:nwc=1:sfr=on:ssfp=1000:ssfq=2.0:smm=sc
    quick.push("lrs+1011_3_nwc=1:stl=90:sos=on:spl=off:sp=reverse_arity_133");
    quick.push("dis-10_5_cond=fast:gsp=input_only:gs=on:gsem=off:nwc=1:sas=minisat:sos=all:spl=off:sp=occ
    quick.push("lrs+1011_5_cond=fast:gs=on:nwc=2.5:stl=30:sd=3:ss=axioms:sdd=off:sfr=on:ssfp=100000:ssfq=
    quick.push("lrs-3_5:4_bs=on:bsr=on:cond=on:fsr=off:gsp=input_only:gs=on:gsaa=from_current:gsem=on:lcm
  }
  else if (atoms > 1200) {
    quick.push("lrs+1011_5_cond=fast:gs=on:nwc=2.5:stl=30:sd=3:ss=axioms:sdd=off:sfr=on:ssfp=100000:ssfq=
    quick.push("dis+1011_8_bsr=unit_only:cond=fast:fsr=off:gs=on:gsaa=full_model:nm=0:nwc=1:sas=minisat:s
    quick.push("dis+11_7_gs=on:gsaa=full_model:lcm=predicate:nwc=1.1:sas=minisat:ssac=none:ssfp=1000:ssfq
    quick.push("ins+11_5_br=off:gs=on:gsem=off:igbr=0.9:igr=1/64:igrp=1400:igrpq=1.1:igs=1003:igwr=on:l
  }
  else {
    quick.push("dis+11_7_16");
    quick.push("dis+1011_5:4_gs=on:gsssp=full:nwc=1.5:sas=minisat:ssac=none:sdd=off:sfr=on:ssfp=40000:ssf
    quick.push("dis+1011_40_bs=on:cond=on:gs=on:gsaa=from_current:nwc=1:sfr=on:ssfp=1000:ssfq=2.0:smm=sc
    ...
  }
}
```

# Results for FOF division of CASC 2016<sup>1</sup>

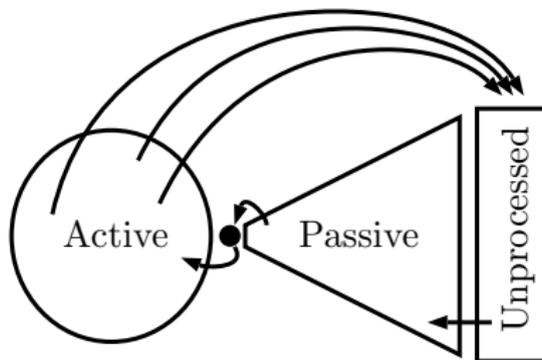


<sup>1</sup>[www.cs.miami.edu/~tptp/CASC/J8/WWWFiles/ResultsPlots.html](http://www.cs.miami.edu/~tptp/CASC/J8/WWWFiles/ResultsPlots.html)

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# The Saturation Loop

Saturate a set of clauses with respect to an inference system



- Initially: the input clauses start in passive, active is empty
- Given clause: selected from passive as the next to be processed
- Move the give clause from active to passive and perform all inferences between clauses in active and the given clause

# The superposition calculus ( )

## Resolution

$$\frac{A \vee C_1 \quad \neg A' \vee C_2}{(C_1 \vee C_2)\theta},$$

## Factoring

$$\frac{A \vee A' \vee C}{(A \vee C)\theta},$$

where, for both inferences,  $\theta = \text{mgu}(A, A')$  and  $A$  is not an equality literal, and  $A$  and  $\neg A'$  are (strictly) maximal in their respective clauses

## Superposition

$$\frac{l \simeq r \vee C_1 \quad L[s]_p \vee C_2}{(L[r]_p \vee C_1 \vee C_2)\theta} \quad \text{or} \quad \frac{l \simeq r \vee C_1 \quad t[s]_p \otimes t' \vee C_2}{(t[r]_p \otimes t' \vee C_1 \vee C_2)\theta},$$

where  $\theta = \text{mgu}(l, s)$  and  $r\theta \not\approx l\theta$  and, for the left rule  $L[s]$  is not an equality literal, and for the right rule  $\otimes$  stands either for  $\simeq$  or  $\not\approx$  and  $t'\theta \not\approx t[s]\theta$

## EqualityResolution

$$\frac{s \not\approx t \vee C}{C\theta},$$

where  $\theta = \text{mgu}(s, t)$

## EqualityFactoring

$$\frac{s \simeq t \vee s' \simeq t' \vee C}{(t \not\approx t' \vee s' \simeq t' \vee C)\theta},$$

where  $\theta = \text{mgu}(s, s')$ ,  $t\theta \not\approx s\theta$ , and  $t'\theta \not\approx s'\theta$

# How important could an ordering be?

Consider proving a formula

$$\psi = \bigwedge_{i=1, \dots, n} (a_i \vee b_i) \rightarrow \bigwedge_{i=1, \dots, n} (a_i \vee b_i)$$

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- goes down to  $3n + 1$  with *Tseitin encoding*:

$$(a_i \vee b_i), \quad (\neg m_i \vee \neg a_i), (\neg m_i \vee \neg b_i), \quad (m_1 \vee \dots \vee m_n),$$

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Question:

What will superposition derive under an ordering where

$$m_i \succ a_j \text{ and } m_i \succ b_j \text{ for every } i \text{ and } j?$$

Orderings typically used in ATPs:

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ATPs typically provide a few schemes for fixing the precedence

## Example

- Vampire: arity, reverse arity, occurrence
- E: frequency (`invfreq`), many more

## Rules of the game

- Fix a single theorem proving strategy in Vampire:  
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- ~12500 solved in 300s by either `casc` or `casc_sat` mode

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Shuffle once:

- $\sim 7100$  solved with a random precedence (3s)
- $\sim 8450$  solved with a random precedence (60s)
- $\sim 9100$  solved with a random precedence (300s)

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- ~7100 solved with a random precedence (3s)
- ~8450 solved with a random precedence (60s)
- ~9100 solved with a random precedence (300s)

Shuffle a few times:

- 9387 solved in a union of 9 independent random precedence 60s runs (1678 problems in the grey zone)

## Question:

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The setup:

- for  $i = 1$  to 100  
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## How many slices could a reasonably good schedule use?

3.0s (7093) 3.0s (330) 3.1s (192) 3.2s (111) 3.3s (101) 4.4s (163) 4.5s (87) 4.8s (79) 5.0s (64) 6.2s (108) 9.6s (156) 11.1s (104) 11.5s (64) 21.4s (169) 205.3s (736)

Solves 9557 problems (9566 on validation set)

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Successfully applied in previous work on literal selection [RSV16]

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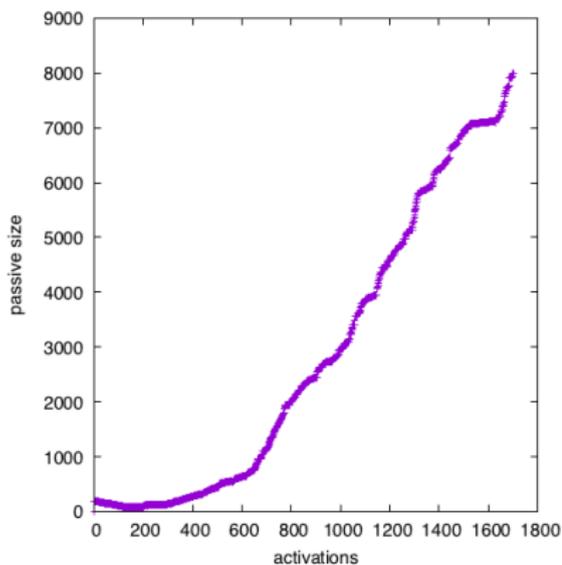
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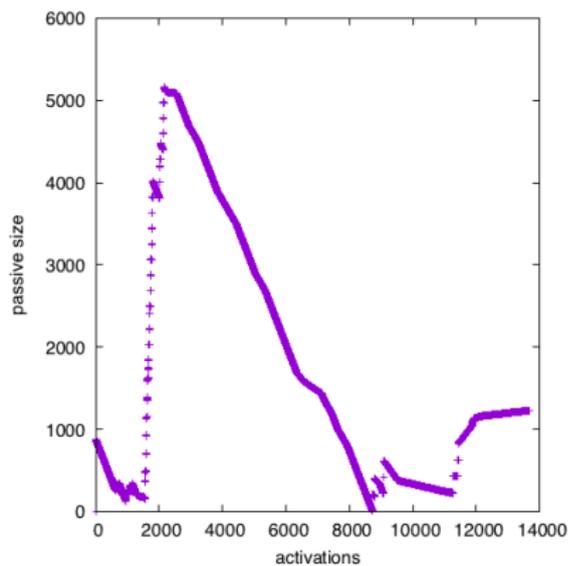
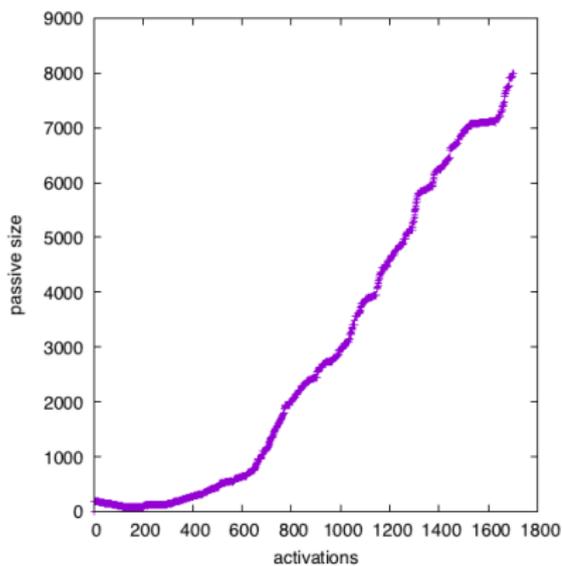
Can this work in practice?

- Probably not under tight time constraints.
- In any case:  
Are there actually any good precedences out there?
- Possible application:  
solve hard previously unsolved problems

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# Can it possibly work?

Using the 9 independent random-precedence 60 second runs

On the set  $P$  of 1678 problems from the “grey zone”

Record size of passive every 100 activations

Compute nine respective sums  $s_i$  until the first stream stops:

$$S_1(p) = s_1(p, 0) + s_1(p, 100) + s_1(p, 200) + \dots$$

...

$$S_9(p) = s_9(p, 0) + s_9(p, 100) + s_9(p, 200) + \dots$$

Denote the average  $S_i(p)$  over (un)successful runs  $i$  as  $\bar{S}_{(un)succ}(p)$

For how many  $p \in P$  is  $\bar{S}_{succ}(p) < \bar{S}_{unsucc}(p)$ ?

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Answer: 1130 (out of 1669)

# How did we evolve, then?

## Optimize\_precedence( $p, t_1, t_2$ )

- run “frequency” for 1s to establish  $act\_cnt$
- spawn a population  $\Pi$  of  $n$  random precedences
- the fitness of  $\pi \in \Pi$  is  $S_\pi(p)$ :  
the sum of the passive set sizes during a run on  $p$   
summing every step from 0 to  $act\_cnt$  activations
- loop for  $t_1$  seconds:
  - pick a  $\pi \in \Pi$
  - randomly (adaptively) perturb  $\pi$  to obtain  $\pi'$
  - evaluate  $\pi'$  as above
  - keep the better of  $\pi$  and  $\pi'$
- Finally, run with  $\pi_{best}$  for  $t_2$  seconds

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How many have solved in total?

- “frequency” 300s: 9457 (40 uniques)
- all the “harmonic” runs: 10030 (202 uniques)
- the long optimizing run: 9604 (87 uniques)
- In total: 10176

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## Future work:

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Thank you for your attention!